Measurement of Large-Signal Parameters of Electrodynamic Transducer

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Abstract: A new method is presented for the dynamic, nondestructive measurement of linear, nonlinear and thermal parameters and the instantaneous state information of woofer, headphones and other electrodynamic actuators. The transducer is measured under normal working conditions while reproducing noise or an ordinary audio signal at high amplitudes. A digital control system estimates the free parameters of the extended electroacoustical model adaptively, identifies the safe range of operation of the particular driver automatically and protects the transducer against thermal or mechanical overload. The identification technique is based on the measurement of the electric signals at the terminals avoiding systematic errors produced by the sensor and the acoustical environment. The identified model is the basis for calculating the transfer behavior and shows the physical causes of nonlinear distortion in the reproduced sound. This information is crucial for assessing loudspeaker quality in the large-signal domain and for improving the loudspeaker design.

1 Introduction

Loudspeakers which sound alike at small amplitudes might behave quit differently in the large signal domain. Objective measurements confirm the dependence of the loudspeaker's transfer function on signal amplitude and the generation of additional harmonic and intermodulation components (distortion) in the reproduced sound. These phenomena can not be explained by the equivalent circuits assuming constant parameters of the lumped elements. Linear modeling and linear parameter measurement are restricted to small amplitudes to be accurate [1-4].

The development of extended models valid in the large signal domain is a topic of electroacoustical research for a long time [5-7]. Thermal and displacement varying parameters have been introduced in the equivalent circuit to describe time-variant and nonlinear mechanisms [8-10]. The resulting model leads to a nonlinear differential equation which has been solved by using the Volterra Series Expansion, perturbation method, harmonic balance and by constructing time periodic maps (Poincare Map) [11–17]. These mathematical tools give valuable insight into the complicated mechanisms and are the basis for numerical simulation of the nonlinear transfer behavior. The progress in physical modeling also initiated the development of nonlinear control systems to compensate actively for signal distortion by preprocessing the electric input signal inversely [18 – 25]. However, all of these powerful techniques are based on an identified model in which the free parameters of the particular transducer are known. A reliable, nondestructive measurement technique is required which performs a fast measurement of the parameters at high accuracy by using a simple and robust equipment which is easy to handle. This paper discusses the different techniques developed so far and presents a new measurement system based on nonlinear control and identification techniques.

2 Physical Modeling

2.1 Lumped Parameter Model

Before starting with the discussion of the measurement techniques the results of extended loudspeaker modeling are summarized by presenting the electromechanical equivalent circuit in Fig. 1 for a voltage driven electrodynamic actuator where u(t) is the driving voltage. The dominant nonlinearities are represented by lumped parameters such as

- b(x) force factor,
- k(x) stiffness of the mechanical suspension and
- L(x) inductance of the voice coil

depending on the instantaneous excursion x of the voice-coil. Since the excursion is a low pass filtered signal, parameter variations are relatively fast generating additional distortion components in the audible band. Variations of the inductance L(x) also generate an electromagnetic driving force $Fm=L_x(x)i^2/2$ where $L_x(x)$ is the first-order derivative of L(x) and I(t) is the voice coil current. A linear but time-variant parameter is

 $R_{\rm e}(T)$ the resistance of the voice coil which depends on the instantaneous voice coil temperature T. Due to the high thermal capacity and resistance of the voice coil and magnet structure [8] the temperature varies slowly

and the variation of the electric resistance has only an influence on the linear transfer behavior (thermal power compression [9]) but does not produce nonlinear distortion. The remaining lumped elements in the equivalent circuit in Fig. 1 are assumed as constants and correspond with the parameters in linear modeling

m mechanical mass of driver diaphragm assembly including voice-coil and air load,

 R_m mechanical resistance of total-driver losses.

2.2 Differential Equation

The relationship in the equivalent circuit yields the set of nonlinear equations

$$u = R_e i + \frac{d(L(x)i)}{dt} + b(x)\frac{dx}{dt}$$

$$b(x)i = m\frac{d^2x}{dt^2} + R_m\frac{dx}{dt} + k(x)x + L_x(x)\frac{i^2}{2}.$$
(1)

written in the general state space form

$$\dot{\mathbf{x}} = \mathbf{a}(\mathbf{x}) + \mathbf{b}(\mathbf{x})u \tag{2}$$

where

 $\mathbf{x}(t) = [x_1, x_2, x_3]^T = [x, dx/dt, i]^T$ is the state vector of the system comprising displacement x, velocity dx/dt of the voice-coil, the electrical input current i and the components of $\mathbf{a}(\mathbf{x})$ and $\mathbf{b}(\mathbf{x})$ which are smooth nonlinear functions of the state vector \mathbf{x} as defined by

$$\mathbf{a}(\mathbf{x}) = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k(x_1)}{m} & -\frac{R_m}{m} & \frac{b(x_1)}{m} + \frac{L_x(x_1)x_3}{2m} \\ 0 & -\frac{b(x_1) + L_x(x_1)x_3}{L(x_1)} & -\frac{R_e}{L(x_1)} \end{bmatrix}$$
(1)

$$\mathbf{b}(\mathbf{x}) = \begin{bmatrix} 0 & 0 & \frac{1}{L(x_1)} \end{bmatrix}^T.$$

Using a smooth, locally defined coordinate transformation $T: x \rightarrow z$ defined by

$$z_{1} = T_{1}(\mathbf{x}) = x_{1}$$

$$z_{2} = T_{2}(\mathbf{x}) = x_{2}$$

$$z_{3} = T_{3}(\mathbf{x}) = -\frac{k(x_{1})x_{1}}{m} - \frac{R_{m}x_{2}}{m} + \left(\frac{b(x_{1})}{m} + \frac{L_{x}(x_{1})x_{3}}{2m}\right)x_{3}$$
(5)

which has also a smooth inverse T^{-1} : $z \rightarrow x$ the general state space in Eq. (2) can be written in the normal state space form

$$z_1 = z_2$$

$$\dot{z}_2 = z_3$$

$$\dot{z}_3 = f(\mathbf{z}) + g(\mathbf{z})u$$
(6)

with

$$f(\mathbf{z}) = f(\mathbf{T}(\mathbf{x})) = -\frac{k_x(x_1)x_1}{m} - \frac{k(x_1)x_2}{m} + \left[\frac{b_x(x_1)x_2}{m} + \frac{L_{xx}(x_1)x_2x_3}{2m}\right]x_3$$

$$-\frac{R_m}{m} \left[-\frac{k(x_1)x_1}{m} - \frac{R_m x_2}{m} + \left(\frac{b(x_1)}{m} + \frac{L_x(x_1)x_3}{2m}\right)x_3\right]$$

$$-\left[\frac{b(x_1) + L_x(x_1)x_3}{mL(x_1)}\right] \left[(b(x_1) + L_x(x_1)x_3)x_2 + R_e x_3\right]$$
(7)

and

$$g(\mathbf{z}) = g(\mathbf{T}(\mathbf{x})) = \frac{b(x_1) + L_x(x_1)x_3}{mL(x_1)}$$
 (8)

In this representation the input/output dynamics is concentrated in a single equation

$$x^{(3)} = f(\mathbf{z}) + g(\mathbf{z})u \tag{9}$$

describing a direct relationship between the electric input voltage and the third derivative of the displacement as discussed in detail in [19-21]. It corresponds with the signal flow chart presented in Fig. 2 which is a convenient basis for system identification. A chain of single order integrators generates the state vector \mathbf{z} which is fed back via the static nonlinear functions $g(\mathbf{z})$ and $f(\mathbf{z})$ to the input where it is multiplied with and added to the voltage u, respectively.

3 Strategies for Parameter Measurement

3.1 Measurement of Linear Parameters

Since the dominant nonlinearities are smooth function of the displacement x, the nonlinear model becomes identical with the linear model if the amplitude of x is sufficiently small. Thus the constant parameters m, R_m and $R_e(T)$ and the nonlinear parameters b(0), L(0), k(0) at the rest position of the voice coil with x=0 can be estimated by standard methods developed for the linear parameters so far. For example, the measurement of the electric input impedance as proposed by Thiele and Small [1 - 3] is a complete dynamic measurement, requires a minimum of additional equipment and provides consistent results as long as the amplitude of the excitation signal is low. However, the linear parameters are only valid at the rest position on loudspeakers and measurements on loudspeakers show that the parameters change by factor 2 and more for higher voice coil excursion in normal operation.

3.2 Measurement of Nonlinear Parameters

The traditional methods which are convenient for the measurements of the linear parameters fails if the transducer behaves nonlinear. The new techniques developed for the measurement of the displacement varying parameters can be classified into static, quasi-static and dynamic methods.

3.2.1 Static and quasi-static Measurement

The static or quasi-static methods [26 – 28] perform a series of small-signal measurements while adding an adjustable dc-offset to the voice-coil excursion and measure the variation of the linear parameters at different working points. After sampling the whole working range this technique results in data points of the nonlinear parameters as a function of the voice coil displacement. Interpolation between the samples may be useful for graphical presentation of the nonlinear characteristics to produce smooth characteristics of the parameter curves as expected from theory. Further analysis can access the results in a loop-up table. Curve fitting based on series expansion may also be applied to the measured samples to reduce the amount of data and to get an analytical expression of the nonlinear parameters.

This sampling technique requires auxiliary means for generating the dc-offset in the displacement. The simplest way is to generate a gravitational force by adding a known mass M_{add} to the diaphragm while the drive-unit axis points in vertical direction. However, this additional mass M_{add} contributes to the moving mass m and changes the dynamic properties of the transducer under test. Alternatively, a dc-current added to the excitation

signal has been proposed to generate an adjustable dc-offset. Measuring the parameters at high excursions where the instantaneous force factor becomes low and the stiffness of the suspension increases, a high dc-current is required which contributes to the thermal load may cause a destruction of the actuator. These disadvantages can be avoided by using a pneumatic pressure as proposed for loudspeakers by Clark [26]. The woofer is mounted in a sealed box which provides a large volume of air and the dc-offset is produced by air pumps with servo control.

If the ac-excursion of the diaphragm at the particular working point defined by the dc-offset is small the actuator can be approximated by a linearized model and straightforward techniques for measurement of the linear parameters can be applied.

3.2.1.1 Measurement of Static Displacement

This technique is the most simple way to measure the stiffness of the suspension at the working point $x = x_{dc}$. A small disturbance such as a small gravitational force is applied to the suspension and the changes of the excursion is measured.

3.2.1.2 Force Balance

The balance technique is also a static technique used to measure the force factor b(x) at the working point $x = x_{dc}$. Again a small disturbance such as a small gravitational force is applied to diaphragm and compensated by the input current to maintain the $x = x_{dc}$.

3.2.1.3 Quasi-static Measurements

Here the well-known dynamic methods developed for the linear parameters at the rest position are applied to the loudspeaker at the working point $x = x_{dc}$. The electric impedance or another transfer function of the loudspeaker is measured by using a small sinusoidal or broad band excitation signal and the linear parameters are derived from the linear loudspeaker model applied to this working point.

All of the static and quasi-static methods showed some disadvantages in practical usage. Sampling the excursion range is time-consuming. During this procedure the actuator is operated under unusual conditions unlike the normal operation of loudspeakers and headphones. Especially, the generating of a high dc-offset in the excursion changes the behavior of the suspension dramatically due to the creep effect. The rest position deviates from the normal place if an extreme dc-offset has been applied to the suspension before. There are also systematic discrepancies between statically or dynamically measured stiffness of the suspension.

3.2.2 Nonlinear System Identification

In order to determine parameter giving the best fitting of the model, the transducer should be operated under normal conditions during the measurement. It would preferable to measure woofers, for example, in the final enclosure by using an excitation signal comparable to an ordinary audio signal. Such a full dynamic measurement can be accomplished by system identification techniques [11, 21, 30 - 32].

3.2.2.1 Volterra Approximation

Inserting the power series expansion of the nonlinear force factor

$$b(x) = \sum_{j=0}^{N} b_j x^j \tag{10}$$

stiffness of the driver suspension

$$k(x) = \sum_{j=0}^{N} k_j x^j$$
 (11)

$$L(x) = \sum_{j=0}^{N} l_j x^j \tag{12}$$

into Eq. 1 we can apply the Volterra series expansion to get an approximative solution of the nonlinear differential equation. The Volterra approach preserves the properties of an ordinary power series but considers the memory of the system which is closely related to the frequency dependency of the system. Thus the precise feedback system presented in Fig. 2 is approximated by a feed-forward model as shown in Fig. 3 comprising a linear, a quadratic, cubic and higher-order homogeneous systems connected in parallel. In the same way as the linear subsystem is characterized by an impulse response, each higher-order subsystem is represented by a kernel which is also a multidimensional weighting function depending on the constant parameters and coefficients of the series expansion in Eqs. 10 – 12. However, the complexity of the analytical expressions grows rapidly with the order and the expansion is truncated after the third-order term in most cases. Transforming the Volterra series in the Laplace domain we get for the linear and each nonlinear subsystem a linear and higher-order system functions. They are a convenient tool for the prediction of the amplitude and phase of harmonic and intermodulation in the output signal generated by a sinusoidal or multi-tone input.

System identification based on the simplified Volterra-model starts with the partial measurement of the kernel or system functions. The kernels are measured by exciting the transducer with a broad-band signal such as gaussian or pseudo-random noise and by applying cross-correlation technique to the input and output signal. The separation of the higher-order kernel from the low-order kernel is more difficult in the time-domain then performing a conventional distortion measurement in the frequency domain by using a multi-tone excitation signal and assigning the amplitude and phase of the harmonic and intermodulation components to the corresponding system functions. Finally, the free parameters in the analytical expressions of the Volterra series representation are estimated by fitting the estimated values to the measured data. This strategy is used in different methods either in a complex way by using special optimization tools [21, 25] or in a simple way by using special conditions during the measurement such as an amplifier with constant current source to use system functions with lower complexity [11].

Unfortunately, the Volterra model is only an approximative solution valid at small amplitudes only where the contribution of the nonlinear subsystems x_{dis} can be neglected in comparison to the linear signal x_{lin} . However, measurements on many loudspeakers show that variation in force factor, stiffness or inductance exceed a ratio of 2 in normal operation and the nonlinear distortion generated by system $f(\mathbf{z})$ and $g(\mathbf{z})$ in Fig. 2 come into the same order of magnitude as the linear input signal u(t) and react on their own generation process. Thus the feedforward structure inherent in the Volterra approach fails and the feedback loop found in the graphical representation of the precise nonlinear differential equation has to be considered in the system identification to avoid systematic errors at large amplitudes.

3.2.2.2 Identification of the Large-Signal Model

The nonlinear differential equation explains the complicated behavior of a nonlinear transducer at large amplitudes such as compression of the fundamental's amplitude at low frequencies, bifurcation about the resonance frequency causing jumping effects and multiple states, instabilities and the dynamic generation of a dccomponents in the displacement. Identification of the nonlinear parameters in the large-signal domain requires a precise model which is closely related to the differential equation. The state space representation in normal form given by Eq. 19 is a good candidate for this approach. There are three different schemes how this model can be connected to the actuator and in which way the identification is performed.

3.2.2.2.1 Parallel Modeling

Fig. 4 shows most natural way. The model represented by the signal flow chart in Fig. 2 is connected in parallel to the real transducer, supplied with the same electric excitation signal and the measured output displacement x is compared with the predicted displacement to produce an error signal used for the optimal adjustment of the free parameters in the model. However, the feedback loop in the model produces some inconveniences. First, the stability of the model can not be ensured for all parameter settings. Secondly, the optimization routine which minimizes a cost function (e.g. least mean squared error signal) may be trapped in a sub-optimal solution since the free parameters are not linear in the error.

3.2.2.2.2 Post-Inverse Modeling

Alternatively, the transducer may be modeled by the inverse model which can be easily derived from the normal form in Eq. 10. Fig. 5 shows the inverse structure connected to the output of the transducer which is just the mirror image of the transducer model where the integrator are replaced by differentiators, the difference between the input signal and function $a(\mathbf{z})$ is divided by $b(\mathbf{z})$. The inverse model with the optimal estimates on the parameter compensates for the entire dynamics of the transducer such that the model output becomes identical with the transducer input u. The inverse model has a consequent feed forward structure behaving stable for all parameters and input signals. Unfortunately, this topology has the disadvantage that any noise corrupting the measurement of the transducer output x(t) causes biased estimates of the parameters [22].

3.2.2.2.3 Pre-Inverse Modeling

Stability of the model and immunity against measurement noise can be obtained by connecting the inverse model to the input of the transducer as shown in Fig. 6. Here the inverse model works as a controller compensating for the linear and nonlinear dynamics of the transducer. If the difference (error) between transducer output x and model input u is minimal the model contains the optimal parameter estimates. However, the free model parameters are not linear in the output error and optimization routine has to consider the properties of the transducer by additional gradient filtering and intermittent updating as proposed in [22].

4 Dynamic Measurement System

The identification technique for the large-signal model is implemented in a new kind of a measurement system for electrodynamic transducers (such as woofers, headphones, shakers) and provides

- linear, nonlinear and thermal parameters and instantaneous state information,
- · dynamic measurement in the large-signal domain,
- on-line operation while reproducing an audio signal,
- · automatic test procedure and easy handling,
- detection of the safe range of operation,
- full protection of the transducer.

Practical realization, additional hardware requirements, the way it is operated by the loudspeaker designer and results of woofer measurements are discussed in this chapter in greater detail.

4.1 Realization

The identification of the transducer model is realized in real time by using digital signal processing to make the measurement as fast as possible and to realize an autonomous system which adapts to the transducer automatically. This system requires minimal input from the user, derives all the required information from the electric signals at the transducer terminals and protects the transducer against thermal and mechanical overload. In this aspect the measurement system is very similar to the nonlinear control concepts developed for woofer systems [24].

Fig. 6 shows the main components of the system comprising a DSP-platform, a power amplifier, a computer with USB interface and if desired an external audio source. The DSP-platform provides some means to measure the voltage and current at the transducer terminals decoupled from the potential of the amplifier output and to convert this signal into the digital domain. The system identification and the remaining functionality is realized inside the DSP. The parameter estimation has been combined with the detection of the back electromotive force (EMF) to dispense with an additional sensor [24]. Using the actuator itself as sensor for monitoring the motional output signal is not only cost-effective but provides accurate and reliable results like the linear methods which are also based on electrical impedance measurement. The identification is not deteriorated by sensor nonlinearities as observed by most displacement lasers and is more independent from the acoustic environment as by using microphones.

The measurement system is capable for generating internally noise used as excitation signal but has also the option for connecting to an external audio source. The noise which corresponds with the simulated program signal as specified in IEC 60268-1 is preferable because it ensures persistent excitation of the loudspeaker and results in faster convergence in the parameter estimation than most of music signals do.

The controller accommodates the gain control unit, a protection system and the feed-forward model in the case of pre-inverse modeling. At the start of the measurement procedure the excitation signal is attenuated to prevent an overload of the unidentified transducer. Slowly the gain of the excitation signal is increased and the voice-coil temperature, the nonlinear parameters and the electric input power are determined simultaneously. The variations of the stiffness is a sensitive indicator for the mechanical load on the suspension system and most woofer systems can handle temporary increases by factor 2 and 4 without any damage. Anyway, if one of the controlled quantities exceeds a pre-defined threshold the gain of the excitation signal is attenuated. The protection system also predicts the maximal displacement and activates an high-pass with variable cut-off frequency to keep the excursion in the safe range and to prevent a mechanical overload. Only general threshold values are defined by the user, the precise range of safe operation is identified for each particular driver automatically.

4.2 Results

During the measurement all of the parameter estimates and the state information of the transducer are transferred to the computer via the USB interface and displayed on the computer. The user knows the instantaneous displacement, velocity and the temperature of the voice coil and the electric signals at the terminals. The identified loudspeaker model makes it possible to measure the amplitude of distortion components produced by nonlinear stiffness, force factor and inductance while reproducing an audio signal. These distortion factors show the instantaneous contribution of each nonlinearity to the total distortion and are the basis to find out the dominant nonlinearity in the particular transducer. The influence of the properties of the excitation signal such as amplitude and spectral characteristics of the music signal can be investigated systematically. In some woofer the peak value of the distortion comes up to 50 % of the amplitude of the total signal.

The measurement can be finished when the parameters are converged to the optimal values and the error signal is minimal. Usually, the relative error in the system identification becomes very small (about 1 %) showing a good fitting of nonlinear model with the real actuator.

Although, most of the large-signal parameters are measured after a few minutes, the thermal behavior and changes of the suspension can be systematically investigated in a long term test. Here the measurement system can be operated as a stand-alone system collecting all of the instantaneous transducer information in a database.

The parameters measured at the beginning of the measurement procedure, when the amplitude of the excitation is still small, correspond with the small-signal parameters determined by conventional techniques. The large-signal parameters measured in the full operation range can be classified into constant, nonlinear and thermal parameters. The nonlinear parameter are presented as the nonlinear curve or as a truncated power series expansion. Fig. 8-10 show as an example, the nonlinear parameters of a woofer used in consumer applications. The inductance in Fig. 9 has a strong asymmetric characteristic which is typical for most drivers. If the voice coil moves towards the back plate the inductance usually increases since the magnetic field generated by the current in the voice coil has a lower magnetic resistance due to the shorter air path. This property can be used for checking the polarity of the loudspeaker and to interpret the direction of the excursion in the diagrams of the nonlinear parameters.

The symmetric decrease in the force factor characteristics corresponds with the dimensions of the voice coil and pole plate. Assuming a symmetric gap configuration and neglecting the contribution of the magnetic field outside the gap a displacement of half the voice coil height generates a decrease of the force factor to 50% of the value at the rest position. If the voice coil height h_V is greater than the gap depth h_G we would expect a region of almost constant force factor for excursions $|x| < (h_V - h_G)/2$ and a steeper decay for larger excursions. In practice the field outside the gap smoothes the characteristic and generates a higher force factor at the rest position in overhang coils and an earlier but more gradual falloff. Most loudspeakers have also an asymmetry in force factor characteristics caused by a sub-optimal rest position of the voice coil and differences of the magnetic field strength above and below the pole plate. Adjusting the voice coil in the gap and correcting the geometry of the pole piece and the pole plate can reduce the second-order distortion significantly without increasing the cost of the transducer.

The nonlinear stiffness characteristics as shown in Fig. 10 describes the behavior of the mechanical suspension under normal working conditions. Some of the variations are caused by geometrical changes of the suspension at large excursions. Modern modeling such finite elements method can explain the macroscopic processes which cause an increase of the stiffness at higher excursion and limit the excursion finally. Most of the loudspeakers like the speaker in Fig. 10 have also an asymmetric stiffness characteristic which is of course unintended and mainly caused by assembling the speaker. Such imperfections can only be detected by measuring the final product. There are also microscopic changes in the impregnated fabric, rubber and foam material which are hard to model. For example the relocation of the fibers in spider materials may cause a phenomenon measured on most drivers: Increasing the amplitude of the displacement the instantaneous stiffness measured dynamically at the rest position k(0) decreases to a lower value. Imposing a high stress seems to

soften the spider for some time but the old properties are restored if the amplitude of the signal is reduced. These behavior has a time constant which is known from the creep effect at very low frequencies and is an interesting subject for further research.

5 CONCLUSION

The nonlinear loudspeaker model and the identified transducer parameters are the basis for assessing the nonlinear transfer behavior. Harmonic and intermodulation distortion can be calculated for any multi-tone excitation signal and time-consuming measurements can be reduced by simulation. The contribution of each nonlinear parameter to the total distortion can be calculated in a distortion analysis showing the dominant nonlinearity of the speaker. The nonlinear parameters also indicate the physical source of the distortion either caused by the design or the assembly of the speaker. The mechanical limits (maximal allowed displacement) and thermal power handling capacity are assessed objectively. The results of loudspeaker diagnosis enables the engineer to optimize the woofer design with respect to asymmetries in the magnetic field, magnet size and weight, ratio of gap depth to the height of the voice coil height and its rest position, dimensions and adjustment of the mechanical suspension.

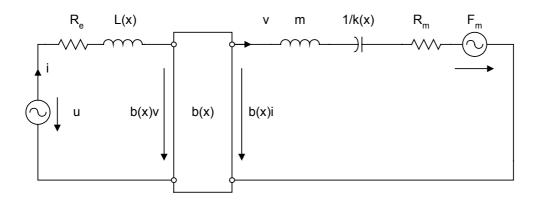
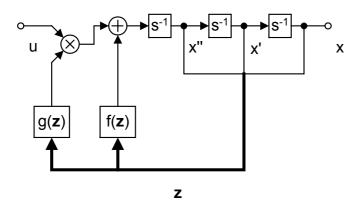


Fig. 1: Nonlinear electro-mechanical equivalent circuit of the transducer.



6 Fig. 2: Signal flow chart of the transducer.

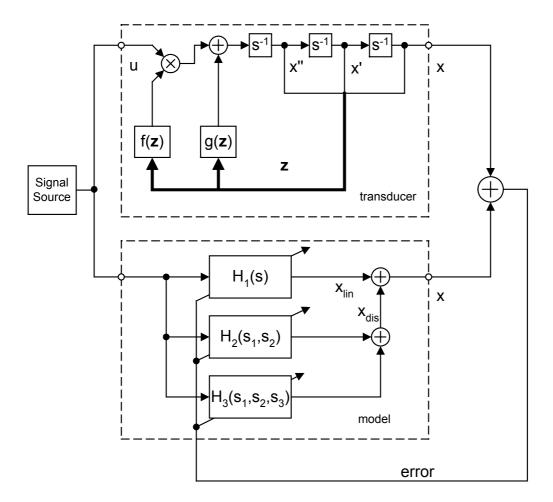


Fig. 3: System identification with Volterra approximation.

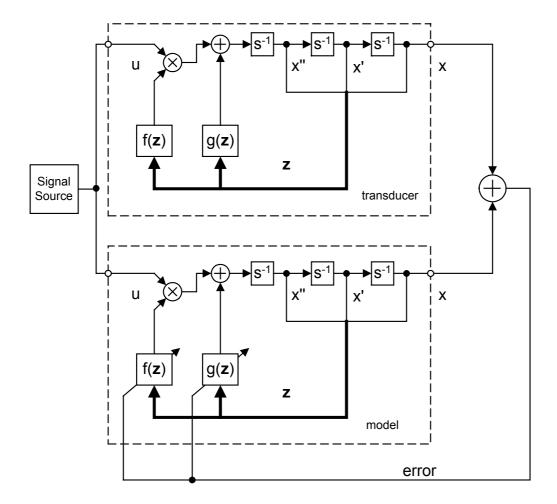


Fig. 4: System identification based on parallel modeling.

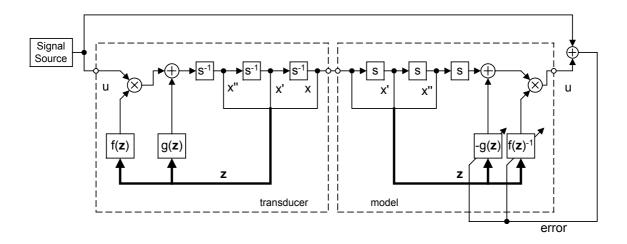


Fig. 5: System identification based on post-inverse modeling.

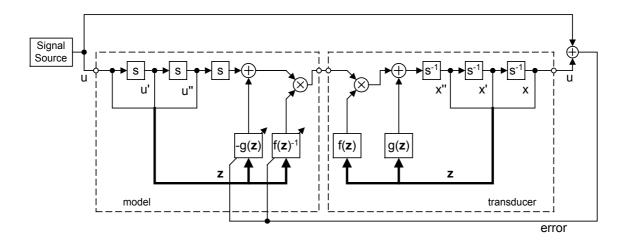


Fig. 6: System identification based on pre-inverse modeling.

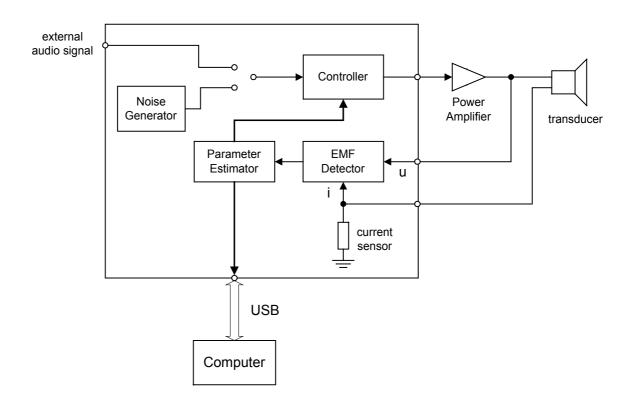
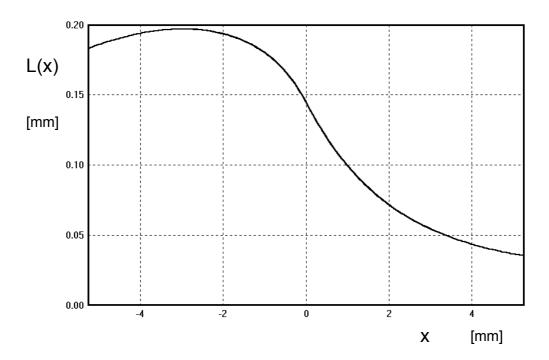


Fig. 7: Novel measurement system for large-signal parameters.



10 Fig. 8: Inductance as a function of voice coil displacement.

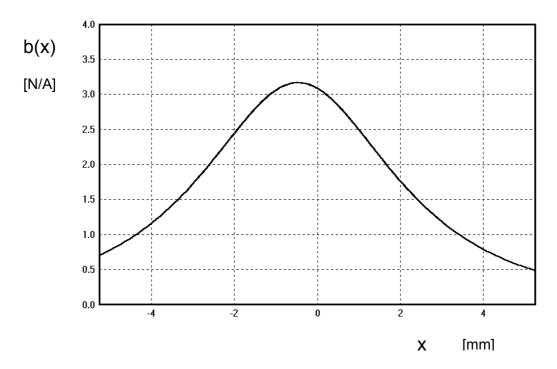
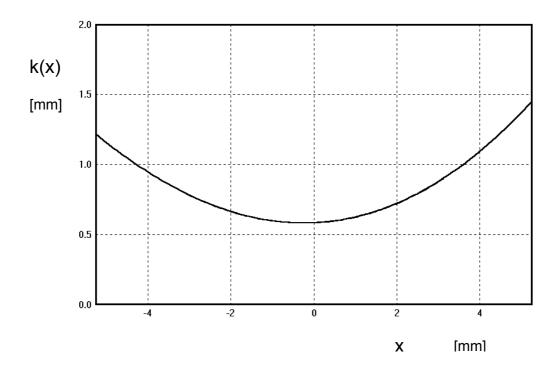


Fig. 9: Force factor as a function of voice coil displacement.



11 Fig. 10: Stiffness as a function of voice coil displacement.

13 REFERENCES

[1] R. H. Small, "Direct-Radiator Loudspeaker System Analysis," J. Audio Eng. Soc., vol. 20, pp. 383 – 395 (1972 June).

[2] R.H. Small, "Closed-Box Loudspeaker Systems, Partl: Analysis," J. Audio Eng. Soc., vol. 20, pp. 798 – 808 (1972 Dec.).

[3] A. N. Thiele, "Loudspeakers in Vented Boxes: Part I and II," in Loudspeakers, vol. 1 (Audio Enginerring Society, New York, 1978).

[4] J. R. Ashley and M. D. Swan, "Experimental Determination of Low-Frequency Loudspeaker Parameters," in Loudspeakers, vol.1 (Audio Enginneering Society, New York, 1978).

[5] R. H. Small, "Assessment of Nonlinearity in Loudspeakers Motors," in IREECON Int. Convention Digest (1979 Aug.), pp. 78-80.

[6] M.R. Gander, "Moving-Coil Loudspeaker Topology as an Indicator of Linear Excursion Capability," in Loudspeakers, vol.2 (Audio Engineering Society, New York, 1984).

[7] A. Dobrucki, C. Szmal, "Nonlinear Distortions of Woofers in Fundamental Resonance Region," presented at the 80th convention Audio Eng. Soc., Montreux, March 4-7, 1986, preprint 2344.

- [8] C. Zuccatti, "Thermal Parameters and Power Ratings of Loudspeakers," J. Audio Eng. Soc., vol. 38, pp. 34 39, (Jan./Feb. 1990).
- [9] D. Button, "A Loudspeaker Motor Structure for Very High Power Handling and High Linear Excursion," J. Audio Eng. Soc., vol. 36, pp. 788 796, (October 1988).
- [10] C. A. Henricksen, "Heat-Transfer Mechanisms in Loudspeakers: Analysis, Measurement, and Design," J. Audio Eng. Soc., vol. 35, pp. 778 791, (October 1987).
- [11] W. Klippel, "Dynamic Measurement and Interpretation of the Nonlinear Parameters of Electrodynamic Loudspeakers," *J. Audio Eng. Soc.*, vol. 38, pp. 944 955 (1990).
- [12] E. R. Olsen and K.B. Christensen, "Nonlinear Modelling of Low Frequency Loudspeakers a more complete model," presented at the 100th convention Audio Eng. Soc., Copenhagen, May 11-14, 1996, preprint 4205.
- [13] M.H. Knudsen and J.G. Jensen, "Low-Frequency Loudspeaker Models that Include Suspension Creep," *J. Audio Eng. Soc.*, vol. 41, pp. 3 18, (Jan./Feb. 1993).
- [14] A. Dobrucki, "Nontypical Effects in an Electrodynamic Loudspeaker with a Nonhomogeneous Magnetic Field in the Air Gap and Nonlinear Suspension," *J. Audio Eng. Soc.*, vol. 42, pp. 565 576, (July./Aug. 1994).
- [15] A. J. M. Kaizer, "Modeling of the Nonlinear Response of an Electrodynamic Loudspeaker by a Volterra Series Expansion," *J. Audio Eng. Soc.*, vol. 35, pp. 421-433 (1987 June).
- [16] W. Klippel, "Nonlinear Large-Signal Behavior of Electrodynamic Loudspeakers at Low Frequencies," *J. Audio Eng. Soc.*, vol. 40, pp. 483-496 (1992).
- [17] J.W. Noris, "Nonlinear Dynamical Behavior of a Moving Voice Coil," presented at the 105th Convention of the Audio Engineering Society, San Francisco, September 26-29, 1998, preprint 4785.
- [18] W. Klippel, "The Mirror Filter A New Basis for Reducing Nonlinear Distortion and Equalizing Response in Woofer Systems," *J. Audio Eng. Soc.*, vol. 40, pp. 675 691 (1992).
- [19] J. Suykens, J. Vandewalle and J. van Gindeuren, "Feedback Linearization of Nonlinear Distortion in Electrodynamic Loudspeakers," *J. Audio Eng. Soc.*, Vol. 43, No. 9, pp. 690-694 (1995).
- [20] W. Klippel, "Direct Feedback Linearization of Nonlinear Loudspeaker Systems," *J. Audio Eng. Soc.*, Vol. 46, pp. 499-507 (1995 June).
- [21] H. Schurer, C. H. Slump, O.E. Herrmann, "Theoretical and Experimental Comparison of Three Methods for Compensation of Electrodynamic Transducer Nonlinearity," *Audio Eng. Soc.*, Vol. 46, pp. 723-739 (1998 September).
- [22] W. Klippel, "Adaptive Nonlinear Control of Loudspeaker Systems," *J. Audio Eng. Soc.* vol. 46, pp. 939 954 (1998).
- [23] F.Y. Gao, "Adaptive Linearization of a Loudspeaker," presented at 93rd Convention of the Audio Eng. Soc., October 1 -4, 1992, San Francisco, preprint 3377.
- [24] W. Klippel, "Nonlinear Adaptive Controller for Loudspeakers with Current Sensor," presented at the 106th Convention of the Audio Engineering Society, Munich, May 8-11, 1999, preprint 4864.
- [25] W. A. Frank, "An Efficient Approximation to the Quadratic Volterra Filter and its Application in Real-Time Loudspeaker Linearization," *Signal Processing*, vol. 45, pp. 97-113, (1995).
- [26] D. Clark, "Precision Measurement of Loudspeaker Parameters," *J. Audio Eng. Soc.* vol. 45, pp. 129 140 (1997 March).
- [27] E. Geddes and A. Philips, "Efficient Loudspeaker Linear and Nonlinear Parameter Estimation," presented at the 91st Convention of the Audio Engineering Society, J. Audio Eng. Soc. (Abstracts), vol. 39, p. 1003 (1991 Dec.), preprint 3164.

- [28] D. Clark and R. Mihelich, "Modeling and Controlling Excursion-Related Distortion in Loudspeakers," presented at the 106th Convention of the Audio Engineering Society, Munich, May 8-11, 1999, preprint 4862.
- [29] D. Clark, "Amplitude Modulation Method for Measuring Linear Excursion of Loudspeakers," presented at the 89th Convention of the Audio Engineering Society, J. Audio Eng. Soc. (Abstracts), vol. 38, p. 874 (1990 Nov.), preprint 2986.
- [30] G. Cibelli, A. Bellini, E. Ugolotti, "Dynamic measurements of low-frequency loudspeakers modeled by Volterra series," in *preprint 4968* presented on 106th Convention of the Audio Eng. Soc., Munich, May 8-11, 1999.
- [31] M. Knudsen, "Estimation of Physical Parameters in Linear and Nonlinear Dynamic Systems, Ph. D. dissertation, Aalborg University, Department of Control Engineering, ISNN 0106-0791, AUC-CONTROL- R93 4010, January 1993.
- [32] M. Knudsen, J.G. Jensen, V. Julskjaer and P. Rubak, "Determination of Loudspeaker Driver parameters Using a System Identification Technique,"," *J. Audio Eng. Soc.* vol. 37, No. 9.